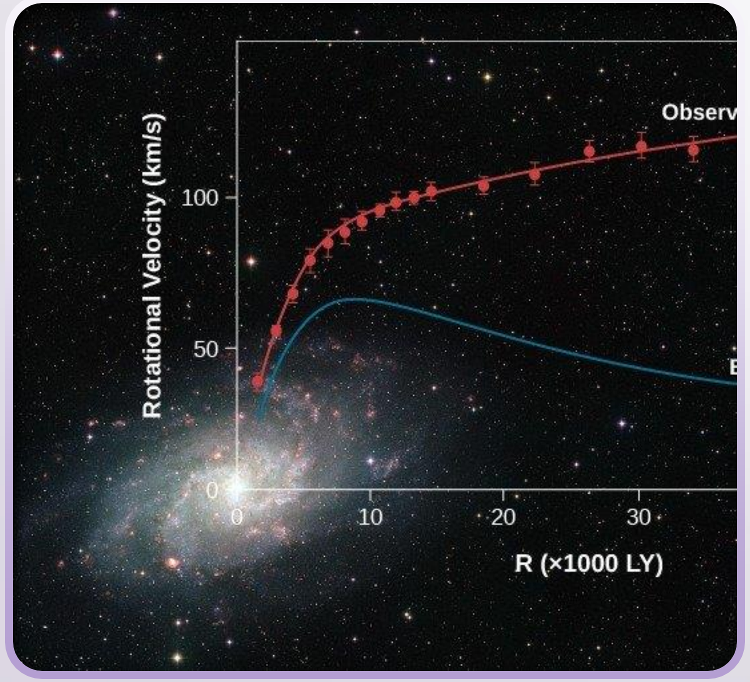

QUANTUM MECHANICAL DESCRIPTION OF DARK ATOMS

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The Dark Atom Model

- The dark atom model is made up of a bound state of a new heavy lepton X^{-2n} and n He^{2+} nuclei.
- In this work, we study the structure of OHe atom since this will give us insight into its properties

Task

- To solve the stationary Schrodinger equation for an isolated OHe atom considering a finite sized He^{2+} nucleus.
- We investigated this problem following the approach proposed by Professor E. Oks in his work with alternative types of hydrogen atoms.

Prof. Eugene Oks' approach

There are 2 known solutions of the Schrodinger equation set up for a particle in a central potential field. These two solutions are characterized by different behavior at small values of r. These are:

$$-\frac{2m}{\hbar^2}\nabla^2\psi(\mathbf{r}) + V(r)\psi(\mathbf{r}) = E\psi(\mathbf{r})$$

$$R(r) \approx r^l$$

The regular solution

$$R(r) \approx \frac{1}{r^{l+1}}$$

The singular solution

Clearly there's a problem with a diverging wave function at r=0 for the singular solution. This solution can still be normalised if we take l=0.

Finite nuclear size

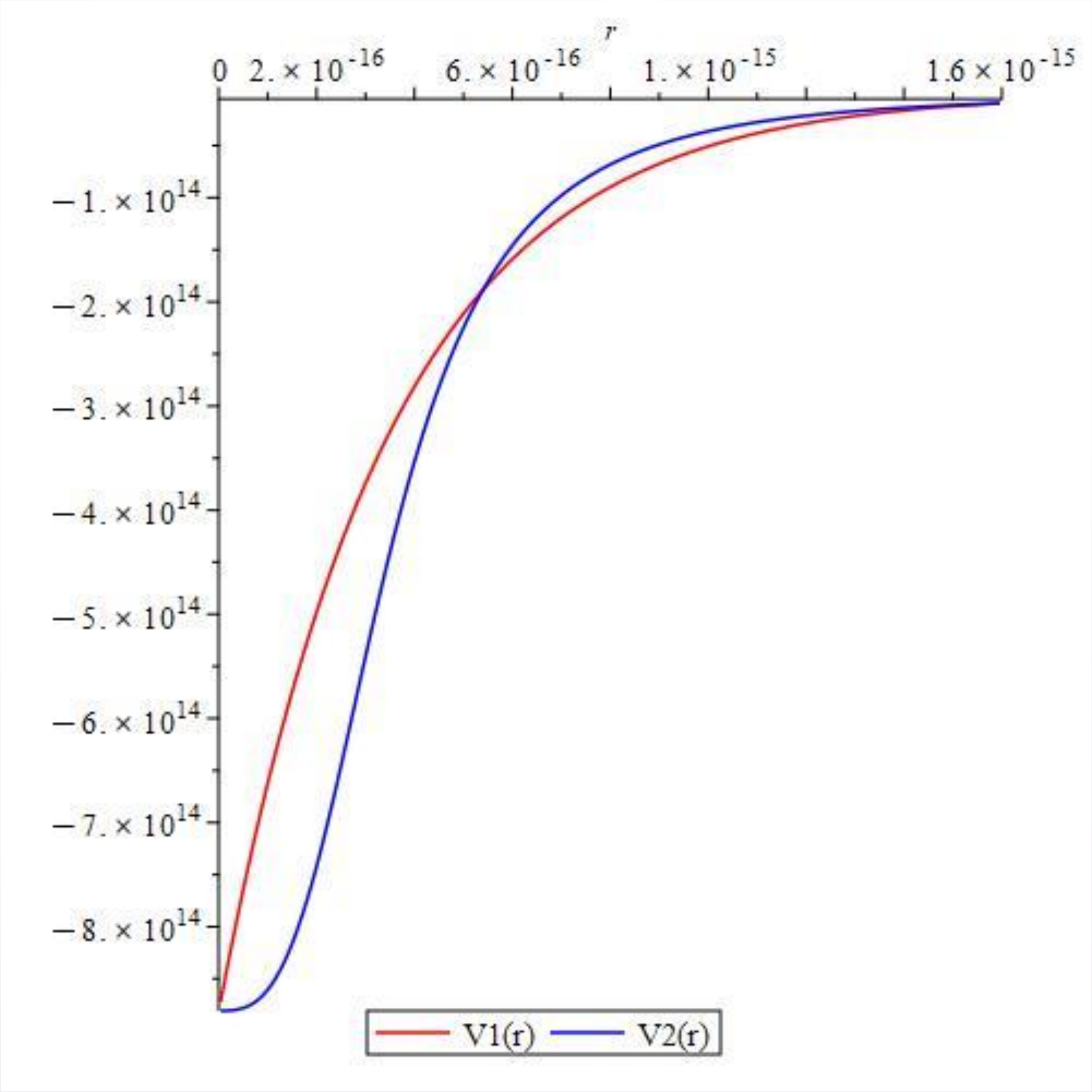
Considering a finite size He^{2+} nucleus and solving a system of equations for the interior and exterior regions allows us to get rid of the divergence at r=0.

Professor Oks showed that the potentials inside the nucleus can be modelled by the following functions:

$$\begin{cases} V(r) = -\left(\frac{Z\alpha}{R}\right) \exp\left[\frac{R-r}{b}\right], & 0 < b \ll R. \\ V(r) = -\left(\frac{Z\alpha}{R}\right) \frac{(R^m+b^m)}{(r^m+b^m)}, & m \geq 3, \quad 0 < b \ll R. \end{cases}$$

Interaction potentials and parameters used

- $R=1.6x10^{-15}$
- m=3
- Z=2
- $b=0.35x10^{-15}$



Solution and analysis

The Schrodinger equation has been written in a spherical coordinate system with the centre coinciding with the position of the particle He^{2+} particle, for the region external to the helium core ($|\mathbf{r}| > R_{(He2+)}$ and for the region located inside the helium core ($|\mathbf{r}| < R_{(He2+)}$ as follows:

$$\begin{cases} \hat{H}_0 \Psi_I = E_{0(He)} \Psi_I, & |\vec{r}| > R_{(He^{2+})}, \\ \hat{H} \Psi_{II} = E_{0(He)} \Psi_{II}, & |\vec{r}| < R_{(He^{2+})} \end{cases}$$

The Hamiltonians:

$$\hat{H}_0 = -\frac{\hbar^2}{2m_{O-}} \Delta - \frac{Z_1 Z_2 e^2}{r},$$

$$\hat{H} = -\frac{\hbar^2}{2m_{O-}} \Delta - \frac{Z_1 Z_2 \alpha}{R_{(He^{2+})}} e^{\left(\frac{R_{(He^{2+})} - r}{b}\right)}$$

The one-dimensional equation in R(r) for the exterior region (exterior to the He^{2+} nucleus)

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dR}{dr} \right) - \frac{l(l+1)}{r^2} R + \frac{2m}{\hbar^2} [E - U(r)] R = 0$$

Solution and analysis

After reducing the equation for the exterior wave function in our system to a one-dimensional radial equation with dimensionless parameter ρ = r/a, and function V (ρ) = R(r)/r as per standard procedure, the problem has been solved in the same way analogous to the solution of the regular exterior solution obtaining:

$$\frac{d^2V(\rho)}{d\rho^2} - \left(\frac{l(l+1)}{\rho^2} - \frac{2Z_1Z_2}{\rho}\right)V(\rho) - 2\varepsilon V(\rho) = 0$$

Asymptotic behaviour of solution:

$$\begin{cases} V(\rho) \xrightarrow{\rho\to 0} \sim c\rho^{-l} \\ V(\rho) \xrightarrow{\rho\to\infty} \sim \rho^{n} e^{-\alpha\rho} \end{cases}$$

We look for a solution in the form:

$$V(\rho) = \rho^{-l}\left(\sum_{k=0}^{n_r} b_k \rho^k\right) e^{-\alpha\rho}$$

Let $g(\rho) = \rho^{-l}\left(\sum_{k=0}^{n_r} b_k \rho^k\right) = \sum_{k=0}^{n_r} b_k \rho^{k-l}$

Solution and analysis

$$V(\rho) = g(\rho)e^{-\alpha\rho}$$

$$V' = g'e^{-\alpha\rho} - \alpha g e^{-\alpha\rho}$$

$$V'' = g''e^{-\alpha\rho} - \alpha g'e^{-\alpha\rho} - \alpha g'e^{-\alpha\rho} + \alpha^2 g e^{-\alpha\rho}$$

Substituting back into the differential equation for V:

$$g'' - 2\alpha g' + \alpha^2 g - \frac{l(l+1)}{\rho^2}g + \frac{2Z_1Z_2}{\rho}g - \alpha^2 g = 0$$

$$g' = \sum_{k=0}^{n_r} b_k(k-l)\rho^{k-l-1}$$

$$g'' = \sum_{k=0}^{n_r} b_k(k-l)(k-l-1)\rho^{k-l-2}$$

Solution and analysis

Substituting back into the differential equation for V:

$$\sum_{k=0}^{n_r} b_k(k-l)(k-l-1)\rho^{k-l-2} - 2\alpha\sum_{k=0}^{n_r} b_k(k-l)\rho^{k-l-1} - l(l+1)\sum_{k=0}^{n_r} b_k\rho^{k-l-2} + 2Z_1Z_2\sum_{k=0}^{n_r} b_k\rho^{k-l-1} = 0$$

$$\sum_{k=0}^{n_r}\left[b_k(k-l)(k-l-1) - l(l+1)\right]\rho^{k-l-2} - \sum_{k=0}^{n_r} b_k\left[2\alpha(k-l) - 2Z_1Z_2\right]\rho^{k-l-1} = 0$$

$$k = 0 \rightarrow -l(-l-1) - l(l+1) = 0$$

Renaming summation indices

$$k' = k-1$$

$$\sum_{k=0}^{n_r} b_{k+1}\left[(k-l+1)(k-l) - l(l+1)\right]\rho^{k-l-1} - b_k\left[2\alpha(k-l) - 2Z_1Z_2\right]\rho^{k-l-1} = 0$$

Solution and analysis

We obtain the iteration for b_k:

$$b_{k+1} = \frac{2\alpha(k-l) - 2Z_1Z_2}{(k-l+1)(k-l) - l(l+1)} b_k$$

$$k = n_r \to b_{k+1} = 0$$

$$\alpha = \frac{Z_1Z_2}{n_r - l}$$

$$\varepsilon = \frac{(Z_1Z_2)^2}{2(n_r - l)^2}$$

$$E_n = -\frac{(Z_1Z_2e)^2}{2a_0(n_r - l)^2}$$

Solution and analysis

We obtain the following form of the radial function:

$$R(r) = C_n r^{-(l+1)} \left(\sum_{k=0}^{n_r} \left(\frac{b_k}{b_0} \right) \left(\frac{r}{a} \right)^k \right) e^{-\frac{Z_1 Z_2}{n_r - l} \frac{r}{a_0}}$$

$$E_n = -\frac{(Z_1 Z_2 e)^2}{2a_0 (n_r - l)^2}$$

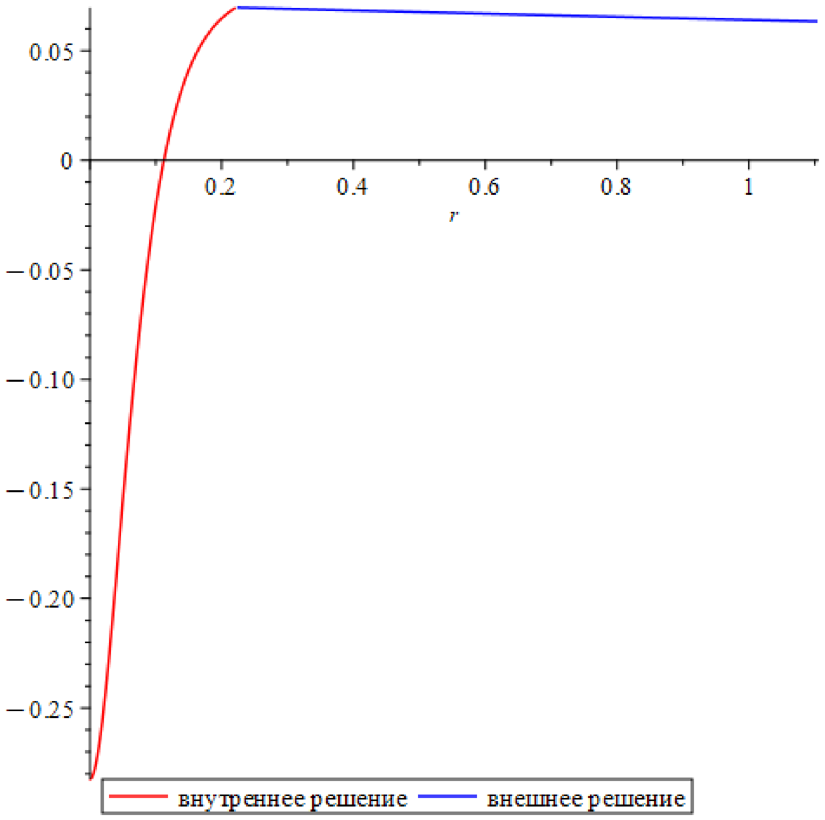
Proceeding as per standard procedure and defining the principal quantum number $n = n_r - l$, we realize that the non-negativity requirement of both l and n_r implies that there is no upper bound for our values of l:

$$n_r \geq 0, n_r = n + l$$

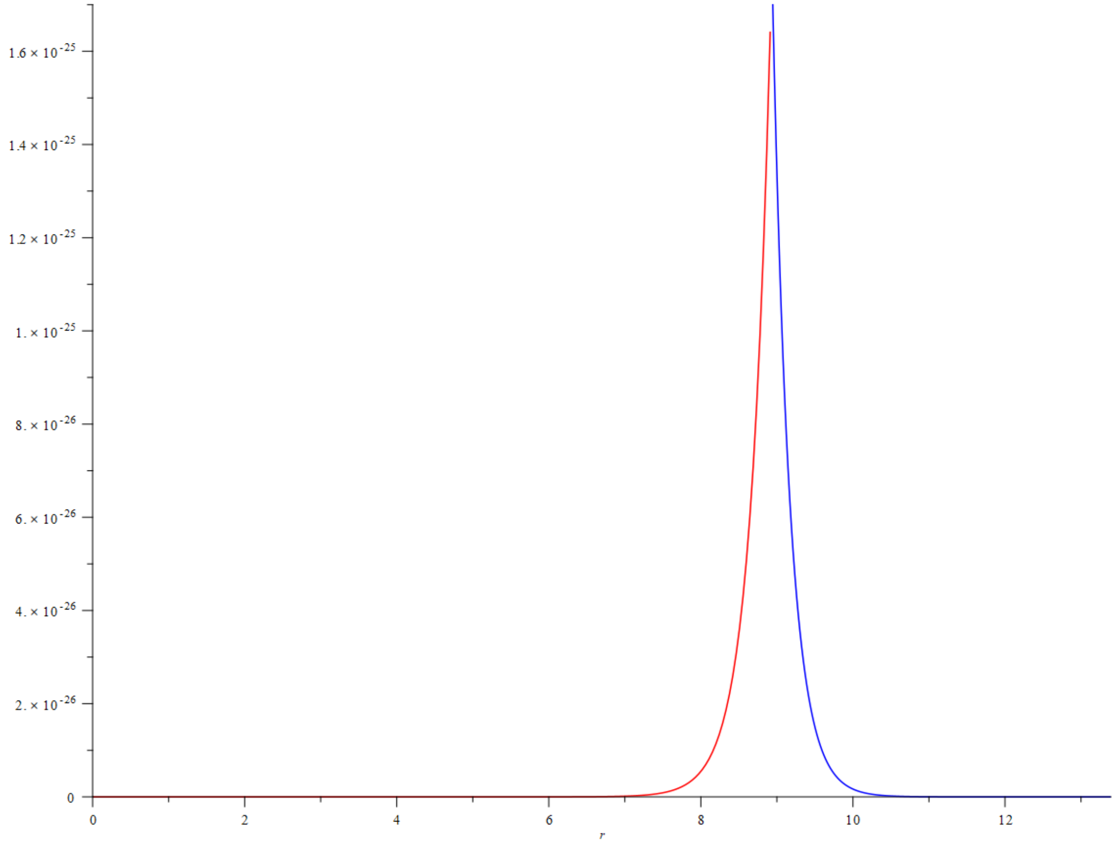
$$n \geq -l \text{ for any } l$$

This result is unphysical. Also noting that this solution is not valid even for l = 0 as suggested by prof. Oks. In his case, the parameters l = 0 and n = 0 result in an infinitely negative energy.

ANALYSIS OF A REGULAR SOLUTION



Wave function of the OHe system



Wave function of O^{2-} in the field of He^{2+}

Conclusion

An attempt to obtain a physically grounded singular solution, following the analogy of Professor Oks, was unsuccessful. The motivation was to obtain a quantum mechanical description of dark atoms, which includes a description of both Bohr structures of the dark atom and structures similar to Thompson structures, without the need to distinguish between them. This remains the goal for future studies.